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Computation of quasi-static coupled fields

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Abstract — **Static electromagnetic fields can be described by partial differential equations of the POISSON type and in the slow time-varying case, the quasi-static field problem, by the diffusion equation. The most common and standard method to solve this type of equations is the finite element method (FEM). Various physical effects, such as ferromagnetic saturation and hysteresis, eddy currents and motional effects can be considered with this method. Looking at the basic equations, typical engineering problems are discussed here, including aspects with respect to coupled quasi-static problems.**

Index terms — **electromagnetic coupling, magneto static problems, magnetic field effects**

I. INTRODUCTION

Most of the physical issues in electrical energy engineering can be described by quasi-static phenomena. Slow varying and periodic fields up to 10kHz are considered to be quasi-static. Electrical energy devices such as electrical motors and actuators, induction furnaces and high-voltage transmission lines are operated by low frequency. Exceptions are microwave devices for electro-heat applications, where inherently the displacement current is not negligible.

II. QUASI STATIC FIELDS

Typical examples of quasi-static electromagnetic fields are the fields excited by coils in rotating electrical machines, in transformers and inductors. Inside these conductors the displacement current is negligible and the magnetic field **H** outside the coil is exclusively excited by the free current current density **J**. For those quasi-static fields, the AMPERE's law is applicable [1].

$$\nabla \times \mathbf{H} = \mathbf{J} \qquad (1)$$

To decide whether the displacement current can be neglected or not, is depending on the wavelength λ of the problem considered in the frequency domain. If it is large, when compared with physical dimensions of the problem l, the displacement current is negligible. To consider this phenomenon in the time domain, the rise time T_a of a step function must be large inside the problem compared to the run time l/v. Field problems are quasi-static if eqs.(2) are true.

$$T_a >> l/v \qquad \lambda >> l \qquad (2)$$

Mainly $T_a \approx 5...10l/v$ respectively $\lambda \approx 5...10l$ is sufficient.

For this class of problem, the interesting fields are slow varying and can be of periodic type.

- static
- slow varying transient
- time harmonic eddy current problems

In time-harmonic problems sinusoidal varying field quantities are assumed. The numerical solution of such problems may be troublesome with respect to the non-linear material modelling [2], [3]. A common numerical approach to consider hysteresis effects, is the PREISACH model [4]. The computation of such material properties may raise difficulties considering transient problems. Here, the hysteresis is time dependent and a curve interpolation must exist during the computation for each instant of time [5].

Assuming low frequencies, the formulation of the 2D electromagnetic problem using the arbitrary vector potential **A** with the ferromagnetic permeability μ, for the static electromagnetic field is

$$\nabla^2 \mathbf{A} = -\mu \mathbf{J} \qquad (3a)$$

respectively for the time domain with the conductivity σ is

$$\nabla^2 \mathbf{A} + \mu\sigma \frac{\partial \mathbf{A}}{\partial t} = -\mu \mathbf{J} \qquad (3b)$$

and in the frequency domain where $\partial \mathbf{A}/dt = j\omega \mathbf{A}$ with the angular frequency ω of the sinusoidal excitation yields

$$\nabla^2 \mathbf{A} + j\omega \cdot \mu\sigma \mathbf{A} = -\mu \mathbf{J} \qquad (3c)$$

For the solution of eqs.(3) the finite element method is in common use. The coefficient matrix of these cases is sparse, diagonal dominant. Therefore, special solver schemes can be used to solve the system of algebraic equations. Effective iterative solver, and in common use, are the incomplete CHOLESKY (IC), symmetric successive overrelaxation (SSOR) pre-conditioned conjugate gradient (CG) methods. Taking advantage of the properties of the system matrix, special algorithms,

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such as algebraic multigrid methods, can be employed to solve large systems very efficiently [6].

The non-linearities can be considered by employing NEWTON-RAPHSON iterations. Material characteristics are given by a list of data samples where cubic interpolating polynoms are used to approximate the data in-between the given samples.

A permanent magnet excitation inside the interesting field domain can be identified as an additional source term $\nabla \times \frac{M_0}{\mu_{PM}}$ for the eq.(3a). The magnet material characteristic is approximated by a straight line $\mathbf{B} = \mu_{PM}\mathbf{H} + \mathbf{M}_0$ determined by the permeability μ_{PM} of the material and its remanence $\mathbf{M}_0$. The assumption of a straight line approximation for this type of material is feasible if modern rare earth materials or ferrites are considered.

To obtain the interesting local field quantities out of the vector potential solution e.g. for the flux density distribution the

$$\nabla \times \mathbf{A} = \mathbf{B} \tag{4}$$

operator can be used.

III. FIELD QUANTITY EVALUATION

To compute forces and/or the local field quantities of a physical problem to e.g. estimate the function and behaviour of a technical device, the FEM is used to solve the problem on a defined domain. The computed field distribution gives information over the operational conditions of the studied device. Those results can be used to optimise the shape of the device, to localise saturation levels and probably leads to changes of the overall construction. Arbitrary conditions such as dangerous fault situations in large electrical machines, or the behaviour of different materials can be simulated by the computer models without the need of an expensive prototype by monitoring the local field quantities.

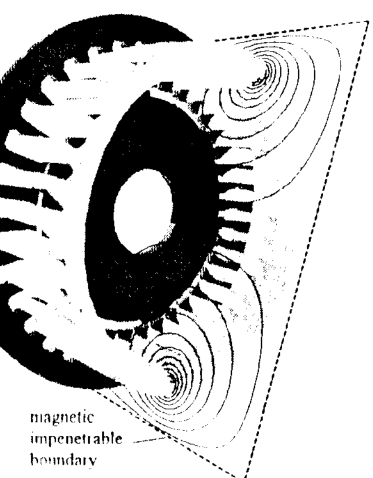

Fig. 1 Field solution on a planar cut in axial direction to compute the end-winding leakage reactance of a permanent magnet servo motor

To simulate the dynamic behaviour of e.g. rotating electrical machines transient time-stepping methods can be used. Such methods can computationally be very expensive. To avoid long-lasting computations very often equivalent models with concentrated elements of the studied device are chosen. Such lumped parameter models are mathematically not as sophisticated when compared to the FEM and therefore are not that computationally expensive, as well as they are less accurate. To overcome this difficulty, the parameters of such an equivalent model can be computed accurately by using the 2D or respectively 3D FEM. As an example, in fig. 1 the 3D FEM model to compute the leakage inductance of a permanent magnet excited servo motor is shown. This element of the lumped parameter model is linear and not depending on the operational conditions of the studied machine. Because of the dimensions of this machine, other parameter such as main reactances can be computed by 2D FEM models.

Until now, only single field types are considered. In reality, field effects are coupled (Fig. 2). The term "coupled problem" is used in many numerical approaches and applications. Various coupling mechanisms in a different context, such as field problems with electrical circuits, methods in a geometrically or physically sense, couplings in time and/or coupled methods to solve a field problem, are meant with this term. In general four classes can be distinguished:

- geometrically or
- physically coupled, or
- hybrid solution method or
- coupled in time.

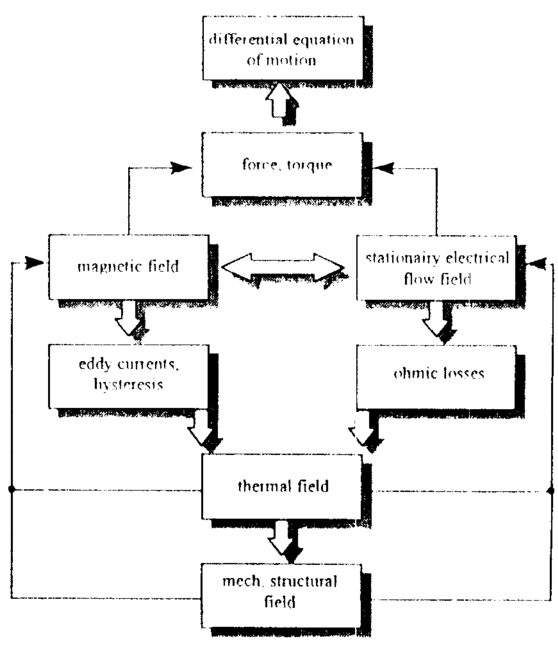

Fig. 2 Simplified structure of coupled fields

The overall term of coupled problems considers the coupled fields and in addition includes the coupling of methods as well. A coupled system or formulation is defined on multiple domains, possibly coinciding, involving dependent variables that cannot be eliminated on the equation level [7]. In the literature, this notion is often linked to a distinguishing context of various physical phenomena or methods, without further specification.

IV. Example

As an example a three phase energy cable represents a thermal-electro-magnetical-electrostatic problem (Fig. 3), where the three static fields are influencing each other by the generated losses. The discretizations, on which the computations for the single field problem are performed, do not have to be identical. Sometimes, only a sub-mesh has a physical meaning: e.g. air carrying a magnetic leakage flux is replaced by a convection constraint in the thermal model; the solid parts can be identical. Even the mesh in areas with more than one continuous degree of freedom can be discretized with different overlapping geometrical meshes and/or element types. Therefore, mesh transition operations have to be defined. The groups of algebraic equations can be solved with a strong coupled or with a cascade coupled strategy.

Here, the results of a three-phase power cable simulation with respect to the coupled magnetic/electrostatic/thermal field problem are shown.

Three phase power cables exist in many variations and types, differing in conductor shape, material choice, conductor arrangement etc. They consist mainly of the following parts, in which several of the previously mentioned loss mechanisms can be found:

- conductor, usually made of copper, suffering from joule losses caused by the high current.
- insulation layers and filling materials, loaded with electric fields and therefore subject to dielectric losses.

Fig. 3 High voltage power cable

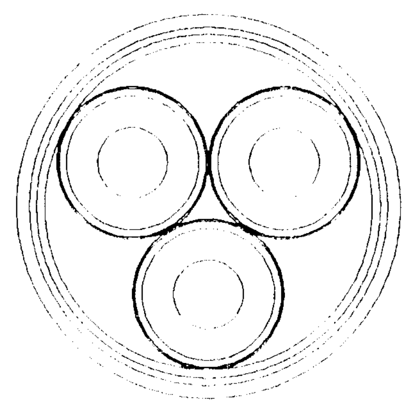

Fig. 4. Geometry of the studied high voltage power cable

- grounded lead sheath around the primary isolation, shielding the electric field; due to its relative low conductance, internal eddy currents can develop.
- mechanical protections (armour), sometimes made of magnetic steel and therefore subject to hysteresis and eddy current losses.

The presence of both, electrically related and magnetically related heat sources leads to a combined model consisting of three field types. Electric, magnetic and thermal field that have to be calculated over a complete or a partial cross-section of the cable and its surrounding.

The Electrical field, described by the scalar potential V is only of interest in the isolation part loaded with an electrical field. Therefore, only a mesh covering this region is required to solve the electrostatic field equations.

The time-harmonic magnetic field is calculated on a larger mesh, since it is only partly shielded by the mechanical protection and thus a leakage field can exist outside the model. This leakage flux is considered by the region surrounding the cable geometry; the far field is modelled by a Kelvin transformed mesh. The losses consist of joule losses in the conducting regions, such as the lead, steel and copper and possible iron losses inside the steel.

The thermal field is represented by the temperature potential T. The static thermal field region consists of the cable with the surrounding soil in which it is buried. From a certain distance, the ground is modelled by a Kelvin transformation and therefore assumed to be infinitely deep. The ground surface is assumed to be cooled by convection.

The losses calculated per element over the previous meshes are projected onto the thermal mesh. The extracted temperatures are used to update the material properties in the other fields. Basically the dielectric loss factor and the thermal conductivity depend on temperature, but this is an effect of minor importance in this example problem. The largest parameter changes are encountered in the conductivity of the copper.

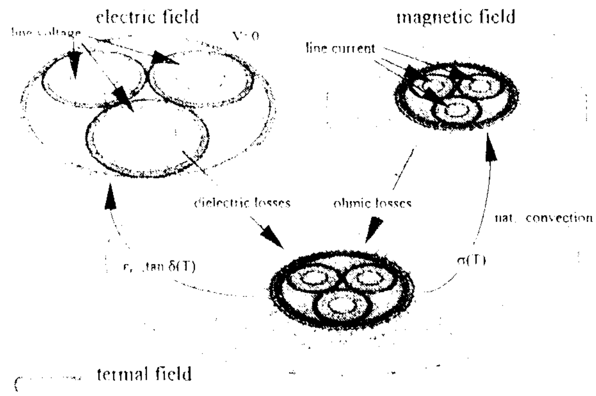

Fig 5. FEM discretizations used for the coupled approach.

To solve the entire problem, a three-domain mesh must be constructed (Fig. 5). The mesh for the electric field contains 9943 first order triangular elements, the magnetic field mesh 15378 and the thermal field mesh 15560 elements.

Here, an iteration and continued up-date of dependent material properties of the single field computations yields the demanded temperature distribution inside an around the high voltage energy cable. The field solutions are linked by projections of one solution to the other problem definition. Due to the different problem properties, the meshes used differ for each problem formulation.

The results of this threefold-coupled problem are collected in the following figures. Open boundary conditions are applied to the magnetic field and to the ground part of the thermal field definition. Dirichlet boundaries can be found in the electrostatic problem and thermal convection at the surface in the temperature field.

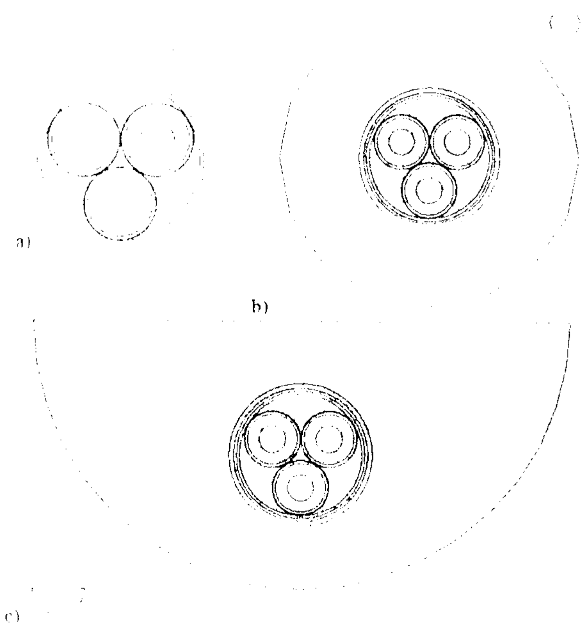

Fig. 6. Solution of the coupled approach: a) electric, b) magnetic field and c) the resulting temperature distribution.

It can be stated that the simulated temperature distribution is in good agreement with measurements. The temperature in the centre of a conductor amounts 84° C. Table I shows the computed values of the heat sources and their location within the models.

TABLE I.
OVERVIEW OF MAIN HEAT SOURCES IN ELECTROMAGNETIC PROBLEMS.

location	loss mechanism	value [W/m³]
copper conductors	ohmic	5,97 10³
conductor isolation	dielectric	1,17 10²
inter-conductor filling material	dielectric	2,55 10²
mechanical protection	ohmic + iron	1,23 10³

V. CONCLUSIONS

An approach to solve a physically coupled thermal/electric/magnetic field problem is introduced. The overall simulation of the problem is performed by a numerically weak coupled cascade algorithm to obtain the steady state temperature distribution in and around a high voltage power cable. Different FEM discretizations are used to simulate the single sub-problems effectively. The different field types are linked by their loss mechanism and thus by up-dating the temperature dependent parameter of the materials applied in successive steps of an iteration. Particular attention must be paid to the convergence of the overall solution. Special relaxation schemes have to be applied to guarantee the fast convergence of the overall solution. The computed results and measurements are in good agreement.

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