Comparison of two methods to determine the d/q-axis lumped parameters of permanent magnet machines with respect to numerical optimisation

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ABSTRACT: In order to manufacture an efficient drive system, the performance of the drive should be optimised in the design stage. Numerical optimisation however, requires the definition of the analysis steps to evaluate a particular design, the definition of the objective function. The ideal objective function would be characterised as accurate and fast. Even so finite element based analysis procedures are described in the literature to a great extent, little attention has been drawn to a comparison of available methods and measurements. Most publications do not give any information about the measurement method used. This paper focuses on the comparison of available methods with measurements and the evaluation of their usefulness for numerical optimisation. The main attention is given to the determination of the d/q-axis inductances.

INTRODUCTION

The analysis of permanent magnet excited electric machines covers a wide range of publications. In recent years, surface mounted permanent magnets dominate, due to the availability of rare earth magnets and the relatively simple requirements on the construction of the rotor design. However, constructions with buried magnets have not lost their attraction, especially when looking at high speed applications were the design provides mechanical advantages. This buried design is also interesting for converter controlled drives, where the influence of the higher harmonic components on the magnets can be limited. The geometry of the rotor constructions varies widely, which limits the applicability of the known analytical methods. The finite element analysis (FEA) provides the tool to tackle different designs with the same analysis tool and formulation. The most accurate analysis, however, will also be the most time consuming. This is contrary of what is desired, when seeking for the combination of FEA and numerical optimisation tools. The construction designer is therefore pressed towards a compromise between accuracy and computational speed of the different algorithms. In this paper two methods to determine the d/q-axis inductances will be compared based on two measured machines. One rotor is equipped with surface mounted magnets, the other one with buried magnets. Figures 1 illustrates the design of the two different machines.

![Fig. 1. a) Type I: unskewed, 6-pole surface-mounted permanent magnet synchronous machine (PMSM) and b) Type II: a 4-pole buried magnet PMSM with a skewed stator.](image)

THE DETERMINATION OF THE D/Q-AXIS INDUCTANCES

There are basically two methods that qualify for the computation of the d/q-axis inductances: the flux linkage method and the loading method. Both allow the analysis of the inductances under load conditions.
IMPROVED FLUX LINKAGE METHOD

The flux linkage method is a known method, where the flux linked with the stator winding is computed via a series of analysis for different rotor positions at constant load. The induced voltage time form can be derived using a Fourier series-representation of the linked flux. The torque angle \( \delta \) can be determined when the geometric rotor and stator d-axis are aligned. However, this is a time consuming process and thus not feasible for numeric optimisations. In order to determine the d-axis inductance, knowledge about the induced voltage from the magnets \( E_0 \) is required. Using \( E_0 \) obtained by a no-load computation introduces a large error, because the saturation of the machine is not considered.

A more elegant algorithm has been proposed in [1], which allows the determination of the linked flux at a number of different rotor positions from only one FEA, but does not provide any information about \( E_0 \) under load. The number of time points of the flux linkage that can be determined is equal to the number of slots in the model. For the motor type I in figure 1, this allows the computation of 6 points, which can be doubled to construct the flux linkage with the reference phase over one electrical period. With the help of a complex discrete Fourier transformation (DFT), it is possible to determine the magnitude and phase angle of the basic harmonic of the induced voltage. The improvement to this method is the determination of \( E_0 \). From the non-linear analysis, the element permeabilities are frozen, and a linear analysis with slightly perturbed excitation currents is carried out. The idea is to apply a linearization around this working point, assuming a change small enough so that there is no change in the saturation level of the machine. The induced voltage \( E_i \) and load angle \( \delta_i \) are determined from this linear solution as well. It is now possible to determine \( E_0 \) under load conditions, thus the d-axis mutual inductance \( L_{md} \) (via the mutual reactance \( X_{md} \)) using the following equations:

\[
E_i \cdot \cos \delta_i = E_0 + I \cdot \cos \beta \cdot X_{md} \quad (1)
\]

\[
E_i^* \cdot \cos \delta_i^* = E_0 + I^* \cdot \cos \beta \cdot X_{md} \quad (2)
\]

Some accuracy aspects are to be discussed in the final paper. Errors are introduced due to the low number of points considered in the DFT and due to round-off errors of the trigonometric functions at low angles.

LOADING METHOD

The loading method [2],[3], appears to be very interesting for optimisation purposes as well, as it requires as well only two field solutions to determine most of the machine parameters. The basic idea is the same as described for the improvement of the flux linkage method using a linearization around the working point. The potential distribution in the air gap is determined from both field solutions. Using a complex fast Fourier transformation, analytic expressions for the flux linkage can be found. The further steps in the analysis are based on the known analytic model of permanent magnet machines. Details concerning the accuracy are discussed in the final paper.

THE DETERMINATION OF THE END WINDING LEAKAGE REACTANCE

The end-winding reactance is a lumped machine parameter that cannot be computed by a 2D field analysis. Two possibilities are feasible: an analytical approach or a 3D numerical analysis. A survey of available literature reveals that a number of formulas have been derived. Applying them to the same machine, the results may differ significantly. This is mainly due to the simplifications that were made during the derivation of the formulas. Most formulations require correction factors or experimentally determined parameters. Another possibility is the computation of the end-winding inductance using the 3D FEA. These inductances are determined by the stored magnetic energy in a 3D model of the end-winding region. Only one magnetostatic analysis is required. Such an arrangement is shown in figure 3. A slice of the iron core is only shown for the orientation, but not present in the analysed model.
MEASUREMENT METHODS OF D/Q-AXIS INDUCTANCES

When comparing measurements with numerical computations, it is always necessary to investigate the appropriate combination of computation technique and measurement technique. A survey of different measurement techniques is presented, with a focus on error sources which have to be considered in the derivation of the d/q-axis inductances.

COMPARISON OF COMPUTATION AND MEASUREMENT

In a first step, the measured no-load inductances (at stand-still) are compared with the computed values (table 1). Here, a good agreement can be found between measured and computed data. However, this good agreement cannot be found anymore in the comparison of the data for the load-tests (table 2, *). Especially the derived d-axis inductances suffer from the impossibility to determine a saturated value of $E_0$. More results from the ongoing measurements will be presented in the final paper.

Table 1. Virtual no-load test (currents 5% of nominal current)

<table>
<thead>
<tr>
<th>machine</th>
<th>voltage/current measurement</th>
<th>step response measurement</th>
<th>flux linkage method</th>
<th>loading method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_d$ [mH]</td>
<td>$L_q$ [mH]</td>
<td>$L_d$ [mH]</td>
<td>$L_q$ [mH]</td>
</tr>
<tr>
<td>I</td>
<td>7.5</td>
<td>11.4</td>
<td>7.8</td>
<td>12.2</td>
</tr>
<tr>
<td>II</td>
<td>180</td>
<td>405</td>
<td>205</td>
<td>410</td>
</tr>
</tbody>
</table>

Table 2. Load test (at equal current amplitude and phase)

<table>
<thead>
<tr>
<th>machine</th>
<th>voltage/current measurement</th>
<th>flux linkage method</th>
<th>loading method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\delta$ [deg]</td>
<td>$L_d$ [mH]</td>
<td>$L_q$ [mH]</td>
</tr>
<tr>
<td>I</td>
<td>6.6</td>
<td>6-10*</td>
<td>14.6</td>
</tr>
</tbody>
</table>

* depending on the choice of $E_0$ under load condition

CONCLUSIONS

The comparison of the results of the computation methods with measurements shows that methods provide good results. The flux linkage method is preferable, as the linked flux is directly computed over the cross section of the winding, thus the winding arrangement is inherently considered. The loading method partially uses the analytical approach by introducing factors such as the winding factors. However, both methods require a monitoring of the error. They are very unreliable for the determination of the inductances if the either d- or q-axis current are very small and not defined if the current component is zero.

ACKNOWLEDGEMENT

The authors are grateful to the Belgian “Fonds voor Wetenschappelijk Onderzoek Vlaanderen” for its financial support of this work and the Belgian Ministry of Scientific Research for granting the IUAP No. P4/20 on Coupled Problems in Electromagnetic Systems. The research Council of the K.U.Leuven supports the basic numerical research.

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