Solving Strong-Coupled Magnetomechanical Systems using Fixed Point Method

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***Abstract*–A strong coupling between the magnetic and the mechanical finite element model is presented. The two coupling terms represent magnetic forces and magnetostriction respectively. The coupled system is solved using a fixed point iteration (successive substitution) with relaxation. The influence of magnetostriction and saturation on convergence is investigated. Convergence is obtained by first solving the problem with linear material characteristic and then gradually increasing the nonlinearity of the material.**

***Index Terms*–coupled magnetomechanical problems, magnetostrictive materials, numerical methods.**

I. INTRODUCTION

The investigation of noise and vibrations of electrical machinery is based upon the coupling between the magnetic field and the mechanical stator deformation. This coupling is usually effected using reluctance forces (Maxwell stress). Since the deformations occurring are small compared to the machine's dimensions, there is no feedback to the magnetic system. Stator deformations are caused not only by reluctance forces, but also by magnetostriction of the stator yoke [1]. Magnetostriction will be the main cause of noise for transformers, inductors and other devices without airgap. The magnetostrictive deformations can be calculated based upon the magnetic field and if these deformations are of the same order of magnitude as the deformations caused by the reluctance forces, again there is no need for feedback to the magnetic system.

However, as soon as magnetostriction becomes important, which is the case in actuators using materials with giant magnetostriction, the magnetic field will be affected and the coupling can no longer be implemented without feedback. The feedback can be provided by using an iterative (weak) solving scheme with the magnetic and mechanical finite element system separated, or the two physical systems can be captured in one magnetomechanical matrix which is solved at once. The latter approach is the one adopted here. Next to the magnetisation characteristic of the iron, the magnetostriction characteristic $\lambda(B)$ is needed. The term *magnetostriction* relates to this $\lambda(B)$ dependency, while the term *inverse magnetostriction* relates to the dependency of permeability on mechanical stress [2]. This latter effect is not considered here but was built into a strong coupling in [3].

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II. THE MAGNETOMECHANICAL SYSTEM

The total energy E of the magnetomechanical system is the sum of the elastic energy U and the magnetic energy W:

$$E = U + W = \frac{1}{2} a^{\mathrm{T}} K a + \frac{1}{2} A^{\mathrm{T}} M A \,, \tag{1}$$

where K is the mechanical stiffness matrix, M is the magnetic 'stiffness' matrix, $a=[u\ v]^{\mathrm{T}}$ is mechanical 2D displacement and A is the z-component of magnetic vector potential. These three unknowns on one node are gathered in one vector $[A\ a]^{\mathrm{T}}$. This suggest the following combination of the magnetic finite element system $MA=T$ and the mechanical finite element system $Ka=R$:

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R \end{bmatrix}, \tag{2}$$

where T is the magnetic source term vector and R represents external forces. The mechanical system is assumed to always stay in its linear range. The coupling term C is related to the magnetic forces (both reluctance forces and Lorentz forces) by

$$F_{mag} = -\frac{\partial W(a)}{\partial a} = -CA = -\frac{1}{2} A^{\mathrm{T}} \frac{\partial M(a)}{\partial a} A \,, \tag{3}$$

for linear magnetic systems and

$$F_{mag} = -\frac{\partial W(A,a)}{\partial a} = -\int_0^A A^{\mathrm{T}} \frac{\partial M(A,a)}{\partial a} dA \,, \tag{4}$$

for nonlinear magnetic systems [4]. The magnetic stiffness matrix M, which is a function of mesh geometry x and material permeability μ, becomes a function of displacement a when $x=x_0+a$ is used instead of $x=x_0$. When magnetostriction is neglected (D=0), then the system (2) can be decoupled into

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R + F_{mag} \end{bmatrix}, \tag{5}$$

and solved using a cascade approach. The coupling term D is related to the magnetostriction effect but cannot capture it completely. The deformation caused by magnetostriction can be represented by a set of magnetostriction forces, as discussed in Section III. The influence of magnetostriction on the magnetic field will be considered in Section IV.

III. MAGNETOSTRICTION FORCES

Magnetostriction is built into the system using a force distribution F_{ms} that is added to R and F_{mag} in (5). By *magnetostriction forces* we indicate the set of forces that induces the same strain in the material as magnetostriction does. This approach is similar to how thermal stresses are usually taken into account [5]. To evaluate thermal stresses, the thermal expansion of the free body (no boundary conditions) is calculated based upon the temperature distribution, and then the thermal stresses are found by deforming the expanded body back into its original shape (back inside the original boundary conditions). To calculate magnetostriction forces, the expansion of the free body due to magnetostriction is found based upon the magnetic flux density, and the magnetostriction forces are found as the reaction to the forces needed to deform the expanded body back into the original boundary conditions.

For finite element models, this can be performed on an element by element basis, where the midpoint of the element (the centre of gravity) can be used as a local fixed point. The magnetostrictive deformation of the element, i.e. the displacement of the three nodes with respect to the midpoint, is found using the element's flux density B^e and the $\lambda(B)$ characteristic of the material, as is explained in detail in [6].

The element's strains λ^e_x and λ^e_y are converted into three nodal displacements $a^e_{ms,i}=(a^e_{x,i}, a^e_{y,i})$, i=1,2,3 considering the midpoint of the element (x^e_m, y^e_m) as fixed:

$$\begin{bmatrix} a_{x,i} \\ a_{y,i} \end{bmatrix} = \begin{bmatrix} x_i - x^e_m \\ y_i - y^e_m \end{bmatrix} \begin{bmatrix} \lambda^e_x \\ \lambda^e_y \end{bmatrix}, \; i=1,2,3 \, , \tag{6}$$

with (x_i, y_i) the co-ordinate of node i. The mechanical stiffness matrix allows us to convert the displacements a^e_{ms} into a set of forces using $F^e_{ms} = K^e a^e_{ms}$. This procedure is performed element by element; it cannot be done for the whole mesh at once, because the displacements a^e_{ms} due to the different elements surrounding a node, cannot be summed. The resulting nodal forces F^e_{ms} however, can be added. As a result, the distribution of magnetostriction forces F_{ms} is obtained:

$$\begin{bmatrix} M & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T \\ R + F_{mag} + F_{ms} \end{bmatrix}. \tag{7}$$

IV. INFLUENCE OF MAGNETOSTRICTION ON THE MAGNETIC FIELD

The coupling term D in (2) is related to magnetostriction. In analogy with the term CA, which is related to the change in magnetic energy W due to a change in displacement, the term Da is linked to the change in elastic energy U due to a change in vector potential (with the corresponding change in magnetic field), but with deformation held constant:

$$Da \sim \frac{\partial U}{\partial A} \tag{8}$$

where ~ instead of = anticipates to the fact that $\partial U/\partial A$ will turn out to have terms independent of a.

Imagine an element with deformation a_0 and flux density B_0. When the flux density in the element increases to $B_0+\Delta B$, the free element expands to $a_0+\Delta a$ due to magnetostriction (no external stresses need to be applied, so ΔU=0). In order to find the elastic energy change ΔU due to ΔB but for constant deformation, the element needs to be shrunk back to its original deformation a_0. The external work done to go back from $a_0+\Delta a$ to a_0 is stored in ΔU and allows us to find an analytical expression for $\Delta U / \Delta B$ in (8).

For a finite element of isotropic material under plane stress, $\partial U/\partial A$ is found analytically to be

$$\frac{\partial U}{\partial A} = \Delta t \, E \, \frac{5/4 - \nu}{1 - \nu} \lambda(A) \frac{d\lambda(A)}{dA} \, , \tag{9}$$

where Δ and t are element area and thickness and E and ν are Young and Poisson modulus. Expanding the area Δ in terms of mesh co-ordinates x_0 and deformation a (linked by $x=x_0+a$) as

$$\Delta(x_0 + a) = x_0^T d_1 x_0 + x_0^T d_2 a + a^T d_3 a \, , \tag{10}$$

expression (9) can be rewritten as

$$\frac{\partial U}{\partial A} = x_0^T D_1 x_0 + x_0^T D_2 a + a^T D_3 a \, . \tag{11}$$

Since the deformation causing noise is much smaller than the size of the finite elements, we have $a << x_0$ and the third term in (11) can be neglected. The first term in (11) does not depend on displacement a and should be put on the right hand side of the coupled system (2). This term can be interpreted as a current density I_{ms} representing the influence of magnetostriction on the magnetic field. Only the second term in (11) can be identified with the term Da in (2). Approximating (11) in this way thus gives

$$\frac{\partial U}{\partial A} \approx I_{ms} + Da + 0 \, , \tag{12}$$

so that the magnetomechanical system (2) can now be filled in completely:

$$\begin{bmatrix} M & D \\ C & K \end{bmatrix} \begin{bmatrix} A \\ a \end{bmatrix} = \begin{bmatrix} T - I_{ms} \\ R + F_{ms} \end{bmatrix}. \tag{13}$$

V. SOLVING THE MAGNETOMECHANICAL SYSTEM

The submatrices M and K are symmetric and positive definite, but the coupling matrices C and D are certainly not each other's transpose image. The magnetic matrix M depends on A due to the nonlinear permeability μ. The dependency of M on a is used only to analytically calculate the partial derivative $\partial M/\partial a$, but is not used to construct the matrix, since $a << x_0$. Using x_0+a instead of x_0 to build the matrix M does not significantly affect the solution of the system. The Young and Poisson modulus E and ν are held constant, so that the mechanical matrix K is constant also. The term C depends on permeability μ but also on its derivative $\partial\mu/\partial B$ because C contains the partial derivative $\partial M/\partial a$ [4].

The magnetomechanical system is highly nonlinear, due to the nonlinearity of the material characteristics $\mu(B)$ and $\lambda(B)$, but also due to the fact that even the derivatives $\partial\mu/\partial B$ and $\partial\lambda/\partial B$ appear. The system (13) can now be solved directly using a fixed point iteration (successive substitution) with relaxation.

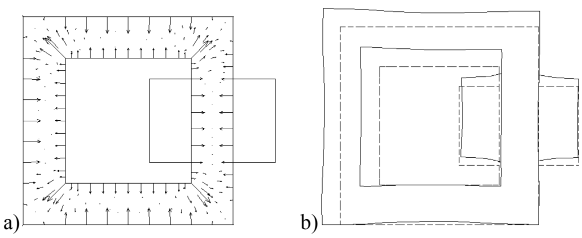

Fig. 1. a) Magnetostriction forces and b) corresponding magnetostrictive deformation for the iron core.

It is possible to construct two different meshes, one specifically suited for the mechanical problem, and one specifically suited for the magnetic problem. However, in the spirit of the strong coupling, and because the magnetic and mechanical unknowns on one node are grouped together, only one mesh is used in this approach. This means that the mechanical properties have to be provided for all materials, including air. In this case, the Young modulus of air was set to E_{air}=1 Pa and its Poisson modulus to ν_{air}=0.1. These values have no physical meaning, but E_{air}=0 cannot be used since that would put zero values on the diagonal of K and destroy its definiteness. E_{air}=1 Pa is small enough compared to E_{iron}=2.10^{11} Pa so that the solution is still valid. However, due to this small stiffness, displacement values inside the air region will not have any physical significance either.

The displacement vector a is scaled using a factor f giving

$$\begin{bmatrix} M & D/f \\ C & K/f \end{bmatrix} \begin{bmatrix} A \\ af \end{bmatrix} = \begin{bmatrix} T - I_{ms} \\ R + F_{ms} \end{bmatrix}, \tag{14}$$

in order to control and equalise the relative order of magnitude of vector potential A and displacement a. If this is not the case, the error estimates used while solving the system may be dominated by the A or the a part of $[A\ a]^{\mathrm{T}}$ and e.g. only the magnetic solution would be valid and the mechanical solution would be inaccurate. This scaling turns out to be important when magnetostriction is important, as explained further on. In order to maintain the elastic energy expression

$$U = \frac{1}{2} a^{\mathrm{T}} K a = \frac{1}{2} (af)^{\mathrm{T}} \frac{K}{f^2} (af), \tag{15}$$

equation (14) is scaled a second time giving

$$\begin{bmatrix} M & D/f \\ C/f & K/f^2 \end{bmatrix} \begin{bmatrix} A \\ af \end{bmatrix} = \begin{bmatrix} T - I_{ms} \\ R/f + F_{ms}/f \end{bmatrix}. \tag{16}$$

VI. CONVERGENCE OF THE FIXED POINT ITERATION: IRON CORE EXAMPLE

The above is illustrated using the model of an iron core excited by a current coil (Fig.1). The core is mechanically constrained by a pin on its bottom right corner, and a horizontal slider on its bottom left corner. The model depth is 1 cm. The core material used in the model is the nonlinear M330-50A. The magnetostriction of the iron core material is assumed to be isotropic and to increase quadratically as a function of flux density, where the value at B=2 T will be used to indicate the severity of the magnetostriction. A typical pattern of magnetostriction forces is shown in Fig.1a and the corresponding magnetostriction deformation is shown in Fig.1b.

Table I. Convergence as a function of magnetostriction for linear magnetic material (μ_r=1000).

magneto-striction λ(2 T) [μm/m]	relaxation factor	number of steps	solution time [s]
0	1.0	5	3.44
2.5	1.0	5	3.61
25	1.0	5	3.41
250	1.0	5	3.38
1000	0.7	9	6.50
1500	0.5	12	8.78
2000	0.3	24	17.10
2500	0.1	67	47.86

The system (16) is solved using a fixed point iteration (successive substitution) with relaxation. As a starting solution, the magnetic system is solved separately using a linear material with μ_r=1000, giving A_0. This needs to be done only once, and the starting solution $[A_0\ 0]^{\mathrm{T}}$ can then be used for all other cases mentioned below. A zero starting solution $[0\ 0]^{\mathrm{T}}$ will not lead to convergence for the fixed point iteration. Inside one substitution step, the matrix system is solved using a GMRES solver and takes about 150 steps for this model.

Table I gives an overview of the relaxation factors used, the number of steps and the CPU time (HP B1000) needed to reach a solution for which the relative change is smaller than 0.1 %. The values in Table I are found using a magnetically linear material, in order to emphasise the nonlinearity coming from magnetostriction. The current in the coil is fixed to 40 A (5.10^4 A/m^2). For low magnetostriction, up to λ(2 T) < 500 μm/m, the convergence is identical to the case without magnetostriction and needs only 5 substitution steps. Most technical materials have magnetostriction below this limit. Only special materials like Terfenol (used in actuators and linear motors based upon magnetostriction) will reach magnetostriction values of the order of 2000 μm/m. The higher the magnetostriction, the lower the relaxation factor than can be used to obtain convergence.

Table II gives an overview of the number of steps needed to obtain a solution as a function of the scaling factor f used in

Table II. Displacement scaling needed to obtain convergence for linear and nonlinear magnetic core material, with and without magnetostriction.

	f	10^{-12}	...	10^{-1}	10^{0}	10^{1}	10^{2}	10^{3}	10^{4}	10^{5}	...	10^{16}
M330	no ms, rel = 0.2	n.c.	n.c.	n.c.	n.c.	n.c.	37	28	35	35	35	35
	λ(2T) = 2.5μm, rel = 0.2	n.c.	n.c.	n.c.	n.c.	n.c.	38	28	35	35	35	35
μ_r=10^3	no ms, rel = 1.0	1	1	1	3	4	7	5	5	5	5	5
	λ(2T) = 1500μm, rel = 0.5	n.c.	n.c.	n.c.	n.c.	n.c.	n.c.	n.c.	n.c.	12	12	12

(16) ('rel' indicates the relaxation factor used, 'n.c.' stands for 'no convergence'). The scaling factor f is of minor importance when the core material is magnetically linear and has no magnetostriction, but is essential when the core material is nonlinear: convergence is obtained only for f=10^2 and higher.

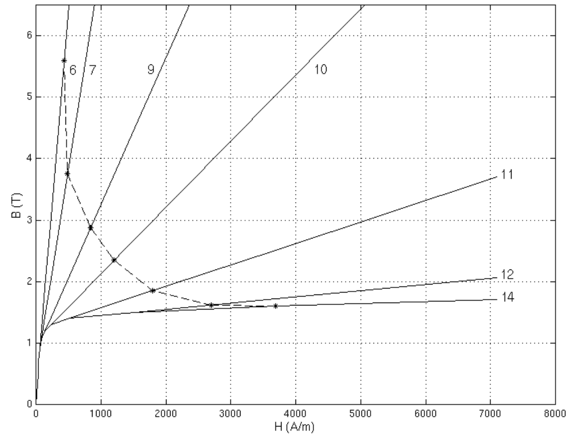

Fig.2 Relaxation of the magnetic material nonlinearity.

For the linear case with high magnetostriction, convergence can only be obtained for f=10^5 and higher.

VII. RELAXING THE MATERIAL NONLINEARITY

The magnetic nonlinearity is the key factor in controlling the convergence of the magnetomechanical system when solved using a fixed point iteration. For high values of the excitation current, the iron will saturate and convergence is obtained by relaxing the material nonlinearity. The magnetic characteristic of the material is defined using 14 samples. Beyond the last point, the finite element code extrapolates the $B(H)$ curve linearly. Fig.2 indicates this extrapolation and the resulting curves when only the first 14, 12, 11, 10, 9, 7 and 6 points of the original $B(H)$ curve are used. These curves do not represent any physical material, but do represent a smooth transition from a linear magnetic material to the nonlinear M330. Using the $[A_0\ 0]^T$ starting solution obtained using μ_r=1000, the problem is solved using material curve 6 (which uses the six first points of the original $B(H)$ curve of M330 and a linear extrapolation beyond H=57 A/m). The solution is indicated with a star on curve 6 in Fig.2. This solution is then used as a starting solution for the same problem but with material curve 7, and so on until the full nonlinearity is taken into account and the final solution is obtained. The trace of the maximum flux density in the intermediate solutions is indicated with a dashed line in Fig.2.

The coil current was fixed to 240 A in order to saturate the entire iron core. Table III shows the CPU time, the number of successive steps needed to obtain convergence and the maximum flux density in the model as a function of how many points of the original $B(H)$ curve were used (relaxation factor = 0.1). The top three lines refer to the case without magnetostriction, while the bottom three lines refer to the case with a magnetostriction of λ(2T)=25 μm/m. Fig.2 indicates the evolution of the maximum flux density for the case without magnetostriction. Magnetostriction will keep the flux density at lower values during the process of relaxing the material nonlinearity. The number of successive steps and the CPU time show no specific tendency, and are not influenced by the presence of magnetostriction. This process of relaxing the material nonlinearity uses a relatively large amount of CPU time, but offers a robust way to convergence.

VIII. CONCLUSION

A numerically strong coupling between the magnetic and the mechanical system has been established. The coupling terms are related to magnetic forces and magnetostriction. Magnetostriction forces are derived based on the analogy with thermal stresses. The magnetomechanical system is solved using a fixed point iteration. The relaxation factors and number of steps needed to obtain a solution strongly depend on the nonlinearities in the system: the saturation characteristic and the magnetostriction characteristic of the materials. A scaling factor for the displacement vector needs to be applied in order to balance the order of magnitude of the elements in the vector of unknowns. The process of relaxing the material nonlinearity is a slow but robust way to ensure convergence, even for highly saturated iron and for relatively high magnetostriction.

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Table III. Number of successive substitution steps, CPU time and maximum flux density in the model during the process of relaxing the magnetic material nonlinearity.

number of points used		*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
number of succ.subst. steps	λ=0	56	62	37	31	37	43	54	49	49	29
CPU time (s)		44	47	29	24	28	32	40	36	36	21
B_{max} (T)		**10.03**	**5.59**	**3.75**	**3.44**	**2.87**	**2.35**	**1.85**	**1.62**	**1.60**	**1.60**
number of succ.subst. steps	λ=25μm/m	61	47	49	34	37	41	54	48	49	31
CPU time (s)		47	36	38	26	28	31	41	35	36	20
B_{max} (T)		**4.54**	**4.26**	**3.51**	**3.28**	**2.81**	**2.33**	**1.85**	**1.62**	**1.60**	**1.60**

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