The Classification of Coupled Field Problems

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Abstract **—The term "coupled problem" is used in many numerical approaches and applications. Various coupling mechanisms in a different context, such as field problems with electrical circuits, methods in a geometrically or physically sense, couplings in time and/or coupled methods to solve a field problem, are meant with this term. For a proper classification of these problems and related solution methods a systematic definition is proposed. It can be used in the evaluation and comparison of solution methods for various problems.**

It must be noted, that the proposed systematic is not complete but can be extended and can serve as the starting point classifying coupled problems in general.

Index terms **— coupled field problems, numerical techniques, finite element method**

I. INTRODUCTION

A coupled system or formulation is defined on multiple domains, possibly coinciding, involving dependent variables that cannot be eliminated on the equation level [1].

In the literature, this notion is often linked to a distinguishing context of various physical phenomena or methods, without further specification. This paper proposes a classification scheme, in which the numerical models, meeting the proposed definitions, can be put. This may lead to the definition of a series of test problems for specified coupled problems and solution algorithms. A classification scheme can simplify the comparison of the various examples and approaches out of the literature, that solve such coupled problems.

Next to "coupled problems" the terms of "weak-" respectively "strong-coupled" will be discussed to propose a more homogenous terminology.

II. COUPLED FIELDS

To start with a definition of standards or a classification of technical physical problems, the properties and the interdependencies of such phenomena must be considered.

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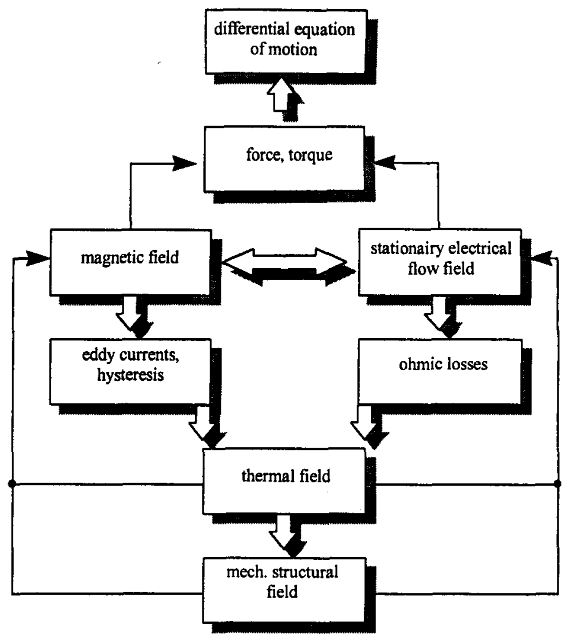

Fig. 1. Simplified structure of coupled field problems.

A general and simplified structure of considered field problems can be taken from Fig. 1. The link between the single fields is determined by material properties depending on the corresponding field quantities. If the field blocks are representing numerical methods to solve the single problem in two dimensions, further couplings to external equations such as electrical circuits, magnetical or thermal equivalent circuit models are possible to complete the scheme.

The link between the drawn blocks is, in the context of coupled problems and its numerical solution, a computer model or method. The following question is, in which way the physical phenomena have to be considered in an overall solution. From the idea of how to link the effects numerically a classification of the methods can be performed.

III. STRONG AND WEAK COUPLING

In general, it is possible to distinguish between the coupled problem in two ways, its physical or in its numerical nature. Very often a coupled is called either

- strong or
- weak.

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In the physical sense, the strong coupling describes effects that are physically strongly coupled and the phenomena can not numerically be treated separately. Respectively, the weak coupling describes a problem where the effects can be separated. The problem with this definition is obvious: If coupled problems are studied, it is not very good known how strong, respectively weak they are physically coupled; this is the requested answer expected from the analysis of the overall problem. For example if the material property describing parameter are non-linearly depending on the field quantities, the coupling, (strong/weak) can change with varying field quantities and the field quantities are the result of the analysis. Therefore, the definition of strong/weak coupling should be chosen with respect to the numerical aspects instead to their physical nature. Choosing for the numerical aspects, it is possible to have a combined strong/weak coupling of field problems. This means that the strategy of coupling can vary during the solution process.

Numerically strong coupling is the full coupling of the problem describing equations on matrix level. The equations of all involved and modeled effects are solved simultaneously. This implies, that the coupling terms are inserted in the coefficient matrix as well.

The numerically weak coupled problem is understood as a cascade algorithm, where the considered field problems are solved in successive steps and the coupling is performed by up-dating and transferring the field depending parameters to the other field definition before solving again.

Since the problems cannot be distinguished by means of elimination, an bi-directional influence exists. The sensitivity of a sub-problem to changes of the variables of the studied problem can differ strongly. It is difficult to quantify a threshold to separate both groups and therefore, the separation may be considered as somewhat subjective.

In this respect, the time constants of the sub-problems play an important role. Usually the thermal and mechanical time constants are several orders larger when compared to the electromagnetic ones. So on a short term, the problem with a larger time constant can be considered as weak coupled. But this is not true if the stationary solution is of interest.

IV. Coupled Problems

The overall term coupled problems considers the coupled fields and in addition includes the coupling of methods as well. The link between different methods to solve a field problem, for example using the combination of finite element and boundary element method is understood as a coupled problem.

Or the classical analytical machine theory delivers models that can be combined with a numerical technique in order to form an overall model (Fig. 2). With respect to computational efforts, for example for dynamic simulations of motor models or observer models for the machine control, the coupling of those methods is advantageous to obtain an accurate but simple overall model of the machine.

Observing problems in the transient modeling of relative motion of machine parts such as in a rotating motor. A possible solution of this modeling problem can be a coupling of geometries by element types with special properties. Overlapping shape functions can be used to join different meshes of a FEM model and this can be seen as a coupled problem as well.

A further example of this type of problems, the coupling of measurements with a numerical model can be given. The basic idea in this type of problem is to measure difficult to obtain parameters and to use them as input parameter for the numerical field computation. Such parameters are mainly non-linearly depending on the field quantities and the interdependency from them is unknown. For example material data obtained by measurements are approximated by interpolating polynoms and can be used in this numerical format during the field computations. Look-up tables with measured data samples are possible as well.

After this first more or less subjective judgment of the various coupling mechanisms, in the following discussion the coupled problems are distinguished with respect to physical and numerical aspects. The single involved field types are described here as sub-problems with specific properties. It will be concluded with a matrix systematic. The matrix entries distinguish between the problem, the model

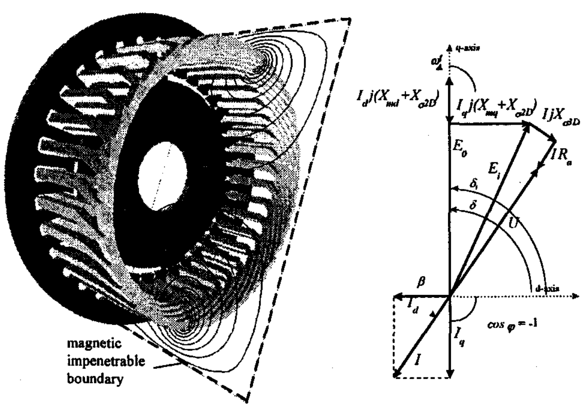

Fig. 2. FEM computation of the end-winding leakage reactance of a permanent magnet servo motor to be coupled with the analytical motor model.

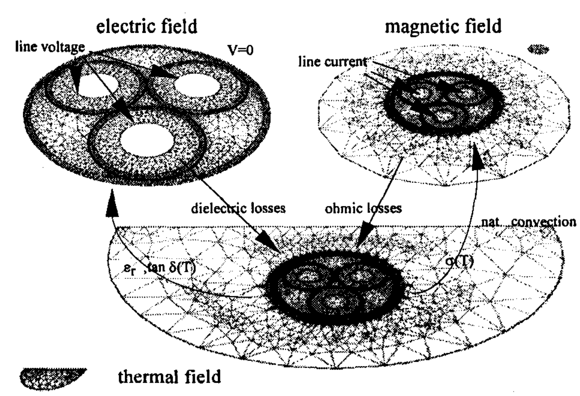

Fig. 3. Coupled FEM electric/magnetic/thermal field problem of a three phase power cable with different meshes for each sub-problem.

description, the coupling mechanism, a proposed iteration scheme and a proposed method to solve the overall field problem.

A. Sub-problem Extent: Domain/Interface

The different interacting physical phenomena described by the coupled problem are defined on partially or totally overlapping domains. For example thermal-electromagnetic problems are belonging to this group (Fig. 3) [4]. For the electro magnetic problem definition the surrounding air has to be modeled. The same domain is considered in the thermal problem by special boundary conditions such as heat transfer due to convection or radiation boundaries. By using the FEM, different meshes for each sub-problem can be used (Fig. 3). The interaction takes place through interface equations. The involved field problems can be numerically strong, i.g. on matrix level, or weak coupled, computed in a cascade algorithm.

B. Sub-problem Discretization Methods: Homogenous/ Hybrid

It is sometimes advantageous to apply different discretization schemes for the sub-problems. The methods used can be different, such as FEM opposed to BEM or FEM

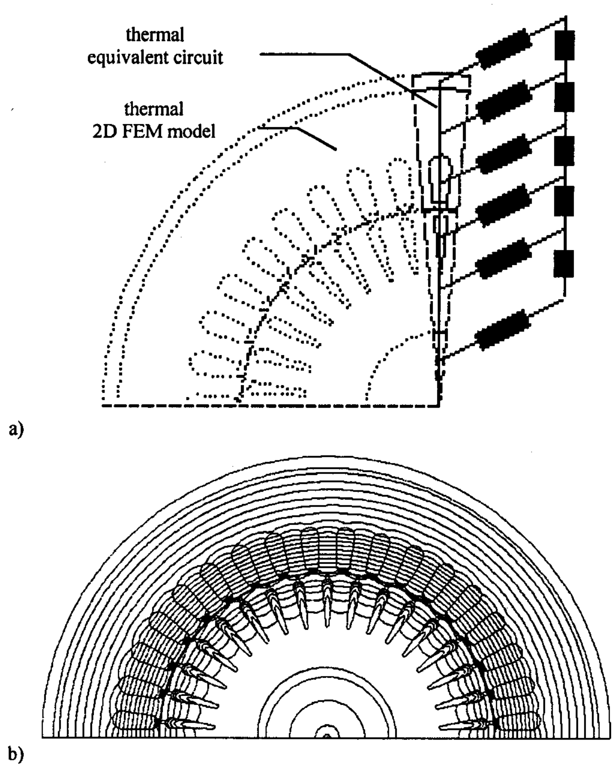

Fig. 4. a) Thermal FEM model coupled with thermal equivalent circuit to consider the axial direction of the heat transfer (circuit elements in radial direction are not drawn) and b) the computed iso-thermal lines.

methods with different types of elements to result in a hybrid method [2]. The discretizations constructed for the sub-problems with a common domain may differ. The addition of algebraic equations originating from equivalent circuit models is possible as well [5]. For example, a two dimensional FEM model to compute the temperature distribution inside an electrical machine (Fig. 4) can be extended by an equivalent thermal circuit model to consider the heat transfer in the axial machine direction. In this way, a quasi three dimensional approach is obtained by the coupled methods. The combination of different FEM models with an additional analytical model is possible. External electric circuits can be coupled to consider the voltage or current driven energy source as well.

C. Numerical Iterative (block) solution methods: full/cascade algorithms

Due to the nature of the physical sub-problems and the chosen discretization method, differing numerical properties can be linked to the equations descending from the sub-problems. A variety of numerical methods can to be chosen to solve the single sub-problem. Most of them can be regarded as block iterative schemes. It is possible to put al the subsystems in a single matrix, with off-diagonal blocks mathematically describing the (linearized) coupling. This can be considered as a numerically strong and thus fully coupled approach.

On the other hand, several blocks can be solved separately with a well-suited equation solver. Not considering possible parallelizations, the solution of the sub-problems is usually obtained in successive steps in a "cascade" algorithm. The newly obtained part of the solution can be used immediately in the next step (Gauss-Seidel-like) or not (Jacobi-like). Other suitable solution techniques are domain decomposition algorithms.

V. Classification Scheme

The made remarks on the classification of coupled problems to build up a matrix systematic, underline the difficulty to put all the mechanisms with respect to their different nature into one system.

The here developed matrix shows couplings between entries in the horizontal as well as in the vertical direction (Table I). Bi-directional links to other entries are possible as well.

The columns of the matrix are representing the mentioned differences of the considered problems with its coupling mechanism. The rows of the systematic are representing the proposed types of problem to put into the appropriate column.

With respect of the geometry, in the first column the studied domains have different properties, such as strong differing material properties. The numerical sub-problems are described by partial differential equations (PDE) and the coupling of the systems of equations is defined by its boundary conditions or interface equations. Depending on the condition of the single sub-problems, a full coupling and weak coupling by cascade algorithms is proposed. For example a hybrid FEM/BEM can be used to solve the overall field problem or in the case of strong differences in the condition of the sub-problems domain decomposition (DD) algorithms, a weak coupling can be employed. Here, an ambivalence of the overall problem can be noticed, using a hybrid method can be considered as a coupled method and the DD as a weak coupling of physical systems.

The physical nature of the field sub-problems is considered in the second column. Examples for this, are coupled magnetic/thermal or other field combinations. The fields can be described either by PDE's or by a combination of PDE and algebraic equations, if equivalent circuit models are used for one of the sub-problems. The coupling is mainly performed by the exchange of the material parameters and source terms (problem Fig. 3) or directly be the circuit equations (problem Fig. 4). For example if external electric circuits are considered. For the solution, numerically strong and weak coupled iteration schemes can be applied.

Hybrid methods are put to the third column. The coupled phenomena have different numerical properties. All possible coupled methods such as FEM, BEM, magnetic- thermal-equivalent circuits as well as the classical analytical field theory coupled to modern numerical techniques are put to this matrix entry. The model description of the overall problem can be done by coupling PDE's, circuit equations, analytical methods (problem Fig. 2) or other methods.

The difference of behavior in time of the coupled effects considers the last column of the matrix. Here, all the transient problems can be found. Simulations with respect to the differential equation of motion, an ordinary differential equation (ODE) are put to this matrix entry. Various, and even coupled methods are suited to solve such in time coupled problems.

CONCLUSIONS

Starting with the various field types and phenomena that are inherently coupled in the physical reality, the terms of coupled problem, methods and fields are defined. The notion of a strong and/or weak coupled mechanism is introduced and motivated by numerical aspects. Examples of coupled problems, methods and phenomena are given to verify and support the statements made. The proposed systematic can probably simplify the further discussions over the numerical solution of "coupled problems" by using the same notations.

It must be noted, that the proposed systematic is not complete but can be extended and can serve as the starting point classifying coupled problems in general.

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TABLE I. CLASSIFICATION SCHEME FOR COUPLED FIELD PROBLEMS.

	geometrically		physically		methods		time	
problem	domain with different properties		different physical properties • magnetical/thermal • magnetical/thermal/dynamical • magnetical/mechanical • electrical/thermal • ...		different numerical properties • FEM • BEM • equivalent circuits • analytical models • ...		different time constants of the coupled phenomena	
model description	PDE/PDE		PDE/alg	PDE/PDE	PDE/PDE, PDE/alg, other couplings		PDE/alg and/or ODE	PDE/PDE and/or ODE
coupling	• boundary conditions • interface equations		• circuit equations	• parameter • source terms	• boundary conditions • interface equations		$\tau_1 \cong \tau_2$	$\tau_1 >> \tau_2$
iteration scheme	full	cascade	full	full cascade	full	cascade	full	cascade
proposed method	FEM/BEM	DD	FEM/circuit equations	FEM/FEM	FEM/BEM	DD, other methods	FEM transient, FEM/BEM transient	FEM/FEM

PDE partial differential equation, alg algebraic equation, ODE ordinairy differential equation, DD domain decomposition method