Influence of Rotor Slot Wedges on Stator Currents and Stator Vibration Spectrum of Induction Machines: A Transient Finite-Element Analysis

Koen Delaere, Ronnie Belmans, and Kay Hameyer

***Abstract*—The stator currents and stator vibration spectrum of a 45-kW four-pole induction machine is computed using a transient two-dimensional finite-element analysis. The analysis is performed for two different rotor geometries: with open and with closed rotor slots. Open rotor slots increase the harmonic content of the air gap field and also the harmonic content of the stator currents. Open rotor slots thus increase the stator vibration level for frequencies higher than 1 kHz.**

***Index Terms*—Rotating machine transient analysis, wedges.**

I. INTRODUCTION

CLOSED ROTOR slots are usually applied to machines up to 250 kW. Larger machines have deep slender rotor bars so that the permeance at the top of the rotor bar (i.e., the bridge permeance) is not that important. Closed rotor slots hold the rotor conductors in place and allow for easier die casting and rotor surface machining. Closed rotor slots are also expected to partially eliminate the rotor slot harmonics, leading to less noise and less iron losses in the stator teeth tips [1], [2]. However, since the rotor slot wedges (or bridges) *saturate* during rated operation, the question remains whether the presence of slot wedges will indeed significantly diminish noise and iron loss. This question is answered here using a transient finite-element analysis of a 45-kW induction machine.

II. TIME-HARMONIC VERSUS TRANSIENT ANALYSIS

Time-harmonic finite-element analysis of rotating electric machines can only capture phenomena at one particular frequency. The slip transformation enables us to find the fundamental electromechanical parameters of the machine. However, for this technique, the magnetic field is still solved for a rotor mesh that is stationary with respect to the stator mesh. Time-harmonic analysis can only correctly incorporate motion effects when the moving parts have a uniform geometry in the direction of motion, e.g., a rail or a slotless rotor made of a single material. As a result, for an induction machine, the time-harmonic analysis with slip transformation cannot capture those field harmonics due to the motional eddy currents in the rotor [3]. Time-harmonic finite-element analysis of rotating electric machines cannot therefore accurately represent the effects of geometrical rotor details, such as open or closed rotor slots, and cannot give any information about the current spectrum.

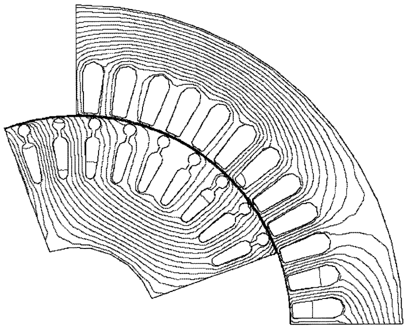


Fig. 1. Magnetic field inside the induction machine for rotor position $\theta = 20^\circ$.

The transient analysis, however, provides the entire harmonic content of the stator current, within the limits of the transient model, e.g., mesh size and time step Δt. Here, assuming a sinusoidal voltage supply, the *transient* analysis is applied to two models of the induction machine with different rotor geometry: with open and with closed rotor slots. This comparison allows us to estimate the influence of rotor slot wedges on the stator current harmonic content and stator vibration spectrum. During steady-state operation, all electromagnetic and mechanical phenomena occur at various fixed frequencies. It is therefore possible to perform a frequency domain analysis based upon a set of samples taken from the time domain and computed using a transient finite element analysis.

III. CALCULATION OF THE STATOR VIBRATION SPECTRUM

For every time step of the transient analysis, the instantaneous magnetic field is used to postprocess the reluctance forces acting on the stator teeth. Fig. 1 shows the flux pattern for one pole of the four-pole induction machine for a certain rotor position. Fig. 2 shows the corresponding reluctance forces acting on the entire stator. The magnetic forces (both Lorentz forces and reluctance forces) are obtained using [4]:

$$\mathbf{F}_{\mathrm{mag}} = -\frac{\partial W}{\partial \mathbf{a}} = -\int_{\mathbf{0}}^{\mathbf{A}} \mathbf{A}^T \frac{\partial \mathbf{M}(\mathbf{A},\mathbf{a})}{\partial \mathbf{a}} d\mathbf{A} \qquad (1)$$

where W represents the magnetic energy and $\mathbf{a}$ represents the displacement or deformation. The deformation is considered in

---

Manuscript received June 18, 2002; revised December 20, 2002. This work was supported in part by the Belgian "Fonds voor Wetenschappelijk Onderzoek Vlaanderen, " the Belgian Ministry of Scientific Research under Grant IUAP P4/20 for Coupled Problems in Electromagnetic Systems, and the Research Council of K.U. Leuven.

The authors are with the Katholic University of Leuven, Leuven, Belgium (e-mail: kay.hameyer@esat.kuleuven.ac.be).

Digital Object Identifier 10.1109/TMAG.2003.810552

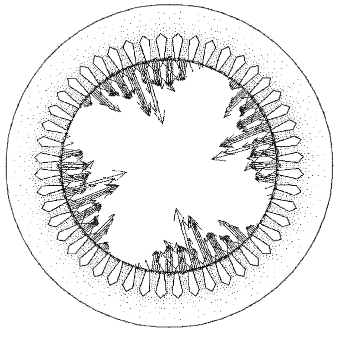


Fig. 2. Force distribution acting on the stator teeth for rotor position $\theta = 20^\circ$ calculated using (1).

the entire model. $\mathbf{MA} = \mathbf{T}$ represents the magnetic finite element system: $\mathbf{M}$ is magnetic stiffness matrix, $\mathbf{A}$ is the vector of nodal values of magnetic vector potential, and $\mathbf{T}$ is the source term representing the current excitation. The condition in this force equation, that the flux has to be kept constant for calculating $\mathbf{F_{mag}}$ from the field energy, is satisfied by keeping the vector potential of all nodes (as it was obtained by the finite-element computation) constant during their displacement $\mathbf{a}$. For the vibration analysis of electric machines, the deformation of the mechanical structure does not signifficantly change the flux pattern. The size of the deformation lies typically in the order of some μm's. In such cases, the deformations of the geometry are so small that it is not required to solve a subsequent geometry modified magnetic problem. However, a rotor eccentricity is an exception to this statement but is not discussed here. In this paper, only the stator deformations are considered. In other words, the magnetic stiffness matrix $\mathbf{M}$ is assumed to only depend on the vector potential $\mathbf{A}$ but not on the deformation $\mathbf{a}$.

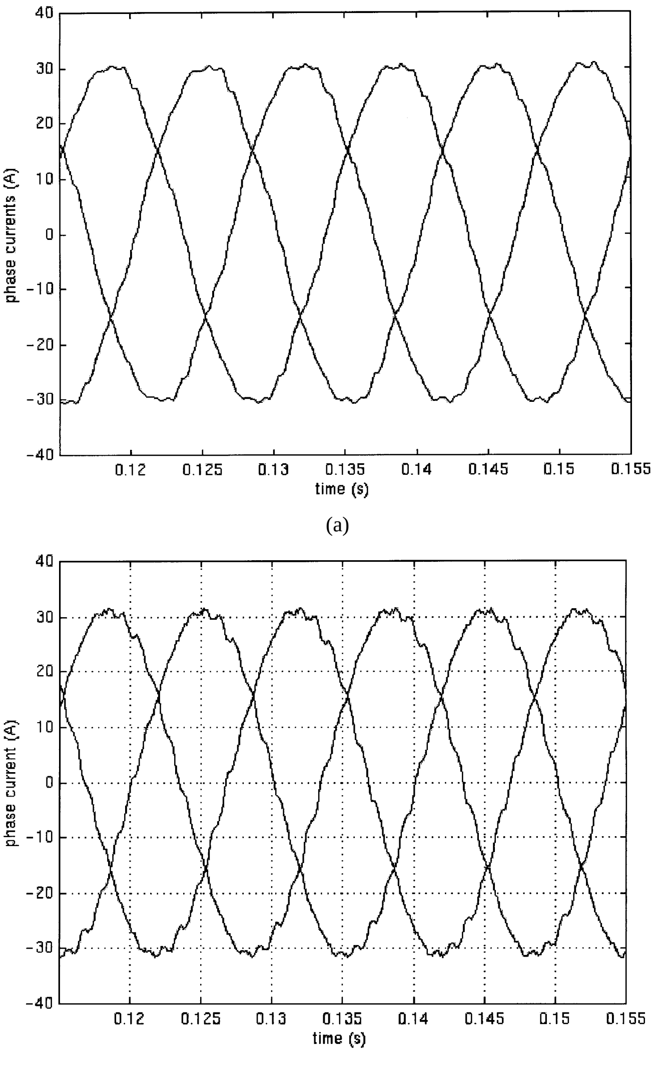
The mechanical finite-element equation of motion of the stator structure is

$$\mathbf{M_m}\ddot{\mathbf{a}} + \mathbf{C_m}\dot{\mathbf{a}} + \mathbf{Ka} = \mathbf{f}(t) \tag{2}$$

where $\mathbf{M_m}$ is the mass matrix, $\mathbf{C_m}$ the damping matrix, and $\mathbf{K}$ the mechanical stiffness matrix. The reluctance force patterns are correlated to the stator mode shapes, giving the mode participation factors. Using the stator mode shapes, the equation of motion (2) is decoupled into a set of modal equations of motions that are then solved separately in the frequency domain. Superposing the modal response spectra then yields the total vibration spectrum of the stator. Details of transforming mode shapes and the reluctance forces acting on the stator teeth into the stator vibration spectrum are given in [4] and [5].

IV. EXAMPLE: 45-KW INDUCTION MACHINE

As a starting solution for the transient analysis, a time-harmonic solution for rotor position $\theta = 0^\circ$ is used. The time step of the transient analysis is $\Delta t = 0.113$ ms which is the time needed for the rotor to turn $\Delta\theta = 1^\circ$. Once the machine is in steady state, $T/\,\Delta t \approx 355$ time steps are used to cover a time span of $T = 0.04$ s. Each time step yields the instantaneous stator currents and flux pattern, which is converted into a reluctance force pattern using (1). In order to speed up the transient analysis, a quality mesh is constructed with as few elements as possible. The magnetic finite element mesh contains 8272 elements and one time step takes approximately 10 s of CPU time on an HP-B1000 workstation. The entire transient analysis thus takes approximately 1 h.



(a)

(b)

Fig. 3. Three-phase stator currents for the induction machine with (a) closed rotor slots and (b) open rotor slots.

The stator winding-scheme of the induction machine can be interpreted as the parallel connection of two smaller machines with a rated current of $I = 23.5$ A for the Y connection, which corresponds to a current amplitude of 33.2 A.

Fig. 3 shows the computed stator currents obtained for the rotor with (a) closed slots and (b) open slots. For open rotor slots, there is more harmonic content in the stator current. Fig. 4 shows the stator vibration spectra obtained for the rotor with closed slots (dashed line) and open slots (solid line). The difference between the two spectra is small below 1000 Hz but substantial for higher frequencies.

From these results it can be concluded that the presence of rotor slot wedges indeed decreases the harmonic content of the stator current and the stator vibration spectrum. Fig. 5(a) shows the rotor slot geometry for closed and open rotor slots. Fig. 5(b) shows a typical flux pattern in the vicinity of the rotor bridge.

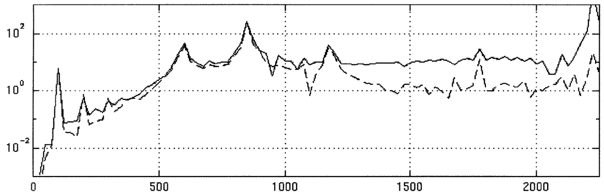
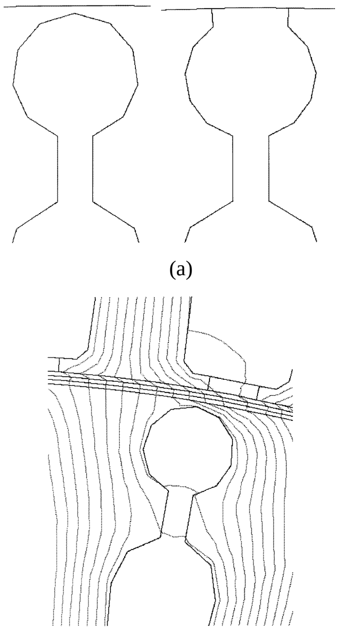


Fig. 4. Stator vibration spectra of the induction machine computed for rotor with (dashed line) closed slots and (solid line) open slots.


(a)

(b)

Fig. 5. (a) Geometry of closed and open rotor slots and (b) a typical flux line pattern in the rotor wedge vicinity.

Investigating the flux density in the vicinity of the rotor slots, it is seen that on the left side of the slot bridge the flux density is relatively small (< 1T), while the right side is in deep saturation. One side of the rotor-bridge does saturate, but the other side does not. As a net effect, the closed slot rotor geometry yields *less* spatial rotor harmonics, leading to a lower level of stator vibration.

V. ROTOR HARMONICS

Rotor harmonics produce field waves in the air gap at the frequencies [1], [6]

$$f_m^{rsh} = f_s \left(m \frac{N_r}{p} (1 - s) \pm 1 \right) \tag{3}$$

where m is an integer, $N_r = 36$ the number of rotor slots, $p = 2$ the number of pole pairs, $s = 0.014\,67$ the slip, and $f_s = 50$ Hz the supply frequency. For $m = 1$, field harmonics at 837 and 937 Hz are found. The corresponding force components are found at the double frequencies 1674 and 1874 Hz as well as at the cross-term frequency 1774 Hz. The corresponding vibration components at 1674 and 1774 Hz are clearly visible in Fig. 4 (dashed line). These discussed rotor harmonics are not captured by a time-harmonic analysis.

VI. CONCLUSION

Assuming a sinusoidal voltage supply, the transient finite-element analysis is used to study the effect of spatial rotor harmonics, i.e., the harmonics due to the slotting of the rotor geometry. The transient analysis is applied to two models of an induction machine with different rotor geometry. One model with open and the other with closed rotor slots (Fig. 5(a)). The comparison of the simulated results allows us to estimate the influence of rotor slot wedges on the stator current harmonics and finally on the stator vibration spectrum. Closed rotor slots partially eliminate rotor slot harmonics, resulting in less noise and less iron losses in the stator teeth tips [1], [2]. When rotor wedges are present, one side of the rotor wedge does saturate, but the other side does not. As a result, the closed slot rotor geometry still gives less spatial rotor harmonics and less stator current harmonics. This leads to a significantly lower level of stator vibrations for frequencies above 1 kHz for the 45-kW induction machine. Below 1 kHz the stator vibration spectrum is comparable for both the closed and open slot geometry. A transient finite-element analysis is necessary to analyze this phenomenon because time-harmonic models do not have the capacity to capture the full harmonic content of the magnetic field.

REFERENCES

[1] T. J. Flack and S. Williamson, "On the possible use of magnetic slot wedges to reduce iron losses in cage motors," in *Proc. Int. Conf. Electric Machines ICEM'96*, vol. 1, Vigo, Spain, 1996, pp. 417–422.
[2] S. Williamson, "Calculation of the bar resistance and leakage reactance of cage rotors with closed slots," *Proc. Inst. Elec. Eng.*, vol. 132-B, no. 3, pp. 125–132, 1985.
[3] H. De Gersem, R. Mertens, and K. Hameyer, "Comparison of stationary and transient finite element simulation techniques with respect to the asynchronous operation modes of induction machines," in *Proc. Int. Conf. Electric Machines ICEM'00*, vol. 1, Helsinki, Finland, Aug. 2000, pp. 66–70.
[4] K. Delaere *et al.*, "Predicting the stator vibration spectrum of induction machines under normal operation," in *Proc. Int. Congr. Exposition Noise Control Eng.*, Fort Lauderdale, FL, Dec. 1999, pp. 6–8.
[5] W. T. Thomson, *Theory of Vibrations with Applications*, Fourth ed. Englewood Cliffs, NJ: Prentice-Hall, 1993.
[6] H. Jordan, *Geräuscharme Elektromotoren*, W. Girardet, Ed: Essen, 1950.